On the Instability of Fractional Reserve Banking

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Introduction

Is fractional reserve banking particularly unstable?

- Yes:
  - Peel's Banking Act of 1844
  - Chicago plan of banking reform with 100% reserve requirement
  - Irving Fisher (1936)
  - Friedman (1959) supported the Chicago plan.
- No:
  - Becker (1956)
  - Adam Smith's the Wealth of Nations (Book II, chapter 2)
- Sargent (2011) summaries the historical debate on this.
- Still on going debate: Switzerland's national referendum of 100% reserve banking in 2018.

This paper

- Focuses on the instability as endogenous cycles (self-fulfilling prophecy)
  - not focusing on banking panic or bank run.
- Constructs a dynamic general equilibrium model of fractional banking by extending Berentsen et al. (2007, JET).
- Establishes the conditions of endogenous cycles, chaotic and stochastic dynamics.

Literature

- Money, credit and banking in the search model:
  Berentsen et al. (2007), Lotz & Zhang (2016), Gu et al. (2013*a*), Gu et al. (2016)

- Fractional reserve banking:
  Freeman & Huffman (1991), Freeman & Kydland (2000), Chari & Phelan (2014), Andolfatto et al. (2019)

- Endogenous fluctuations, chaotic dynamics, and indeterminacy:
  Baumol & Benhabib (1989), Azariadis (1993), Benhabib & Farmer (1999) Gu et al. (2013*b*), Gu et al. (2019)

Environment

- Time, goods
- Agents, banks, and the central bank
- Preferences

Environment

- Time, goods
  1. $t = 0, 1, 2..., \infty$
  2. Each period has three subperiod:
     - Centralized Goods Market (CM)
     - Centralized Financial Market (FM)
     - Decentralized Market (DM): bilateral trade, subject to anonymity, limited commitment
  3. Perishable DM/CM goods.
- Agents, banks, and the central bank
- Preferences

Environment

- Time, goods
- Agents, banks, and the central bank
  1. Agents: measure 1; maximize life time utility;
     with prob σ, buyer, with prob $1-\sigma$, seller in the DM;
     DM types are realized in the FM.
  2. Banks accept deposit and lend loan.
  3. The central bank control money supply M_t via lump-sum tax/transfer. Let γ money growth rate, $\gamma = M_t/M_{t-1}$
- Preferences

Environment

- Time, goods
- Agents, banks, and the central bank
- Preferences
$$U(X) - H + u(q) - c(q)$$
  - CM consumption X; CM disutility for production H; DM consumption q; discount factor: β
  - efficient DM consumption, q^* solves $u'(q^*) = c'(q^*)$.

CM problem

$$W_t(m_t, d_t, \ell_t) = \max_{X_t, H_t, \hat{m}_{t+1}} U(X_t) - H_t + \beta G_{t+1}(\hat{m}_{t+1})$$

$$\text{s.t. } \phi_t \hat{m}_{t+1} + X_t = H_t + T_t + \phi_t m_t + (1 + i_{d,t})\phi_t d_t - (1 + i_{l,t})\phi_t \ell_t \quad (1)$$

- Standard results: $W_t(m_t, d_t, \ell_t)$ is linear in m_t, d_t, and ℓ_t
- FOC for $\hat{m}_{t+1}$:

$$\phi_t = \beta G'_{t+1}(\hat{m}_{t+1}) \quad (2)$$

DM trade

- DM value function for buyer

$$V_{b,t}(m_t - d_{b,t} + \ell_{b,t}, d_{b,t}, \ell_{b,t}) = \alpha[u(q_t) - p_t] + W(m_t - d_{b,t} + \ell_{b,t}, d_{b,t}, \ell_{b,t})$$

where $p_t \leq m_t - d_{b,t} + \ell_{b,t}$.

- DM value function for seller

$$V_{s,t}(m_t - d_{s,t} + \ell_{s,t}, d_{s,t}, \ell_{s,t}) = \alpha_s[p_t - c(q_t)] + W_t(m_t - d_{s,t} + \ell_{s,t}, d_{s,t}, \ell_{s,t})$$

Trading Mechanism | Derivatives of DM value functions

DM trade

- A general trading mechanism $p = v(q)$, where $p \leq z$ (Gu & Wright 2016). $v'(q) > 0$
- Let p^* be a payment to get q^*.
- Terms of trade are given by

$$p = \begin{cases} z & \text{if } z < p^* \\ p^* & \text{if } z \geq p^* \end{cases} \qquad q = \begin{cases} v^{-1}(z) & \text{if } z < p^* \\ q^* & \text{if } z \geq p^* \end{cases}$$

- $\lambda(q) = u'(q)/v'(q) - 1$ if $p^* > z$ and $\lambda(q) = 0$ if $z \geq p^*$

Trading Mechanism Derivatives of DM value functions

FM problem

- Types are realized at the FM.

$$G_t(m) = \sigma G_{b,t}(m) + (1-\sigma) G_{s,t}(m) \tag{3}$$

- Type-j agent solves the following problem

$$G_{j,t}(m) = \max_{d_{j,t},\ell_{j,t}} V_{j,t}(m - d_{j,t} + \ell_{j,t}, d_{j,t}, \ell_{j,t}) \quad \text{s.t} \quad d_{j,t} \leq m \tag{4}$$

where $j \in \{b, s\}$

- FOCs are:

$$\frac{\partial V_{j,t}}{\partial \ell_{j,t}} \leq 0 \tag{5}$$

$$\frac{\partial V_{j,t}}{\partial d_{j,t}} - \lambda_d \leq 0 \tag{6}$$

where λ_d is the Lagrange multiplier for $d_{j,t} \leq m$.

Bank's problem

- A representative bank accepts nominal deposit and lends nominal loan.
- The bank maximizes profit

$$\max_{d,\ell} \quad (i_l \ell - i_d d) \quad s.t. \quad \chi \ell \leq d \tag{7}$$

- FOCs are

$$0 = i_l - \lambda_L \tag{8}$$
$$0 = -i_d + \lambda_L / \chi \tag{9}$$

- For $\lambda_L > 0$, we have

$$i_l = \chi i_d$$

Definition of equilibrium

Given (γ, χ), an equilibrium consists of the sequences of

- prices $\{\phi_t, i_{l,t}, i_{d,t}\}_{t=0}^{\infty}$,
- real balances $\{m_t, \ell_{b,t}, \ell_{s,t}, d_{b,t}, d_{s,t}\}_{t=0}^{\infty}$, and
- allocations $\{q_t, X_t, \ell_t\}_{t=0}^{\infty}$ satisfying the following:
  - Agents solve CM and FM problems: (1) and (4)
  - A representative bank solves its profit maximization problem: (7)
  - Markets clear in every period:
    1. Deposit Market: $\sigma d_{b,t} + (1-\sigma)d_{s,t} = d_t$
    2. Loan Market: $\sigma \ell_{b,t} + (1-\sigma)\ell_{s,t} = \ell_t$
    3. Money Market: $m_t = M_t$

Equilibrium

Proposition

Given (γ, χ), an equilibrium can be summarizes into the following difference equation:

$$z_t = f(z_{t+1}) \equiv \frac{z_{t+1}}{1+i}\left[\frac{1-\sigma+\sigma\chi}{\chi}\alpha L(z_{t+1}) + 1\right] \qquad (10)$$

where $1+i \equiv \gamma/\beta$, $z_t = \phi_t m_t(1-\sigma+\sigma\chi)/\sigma\chi$, and $L(z) \equiv \lambda \circ v^{-1}(z)$ is liquidity premium.

Stationary Equilibrium

Cycles

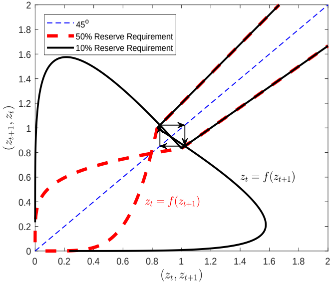
$$z_t = f(z_{t+1}) \equiv \underbrace{\frac{z_{t+1}}{1+i}}_{\text{increasing in } z_{t+1}} \underbrace{\left[\frac{1-\sigma+\sigma\chi}{\chi}\alpha L(z_{t+1}) + 1\right]}_{\text{decreasing in } z_{t+1}}$$

- $f(z_{t+1})$ is generally nonmonotone.
- If the second term dominates the first term, we can have $f'(\cdot) < -1$ which is a standard condition for the existence of cyclic equilibria

Proposition (**Monetary Cycle**)

If $f'(z_s) < -1$, there exist a two-period cycle with $z_1 < z_s < z_2$.

Cycles



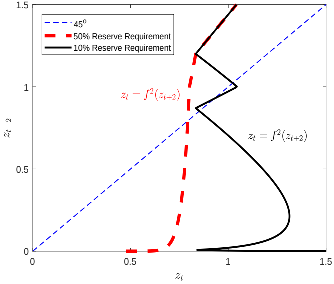


Figure 1: A Two-period Cycle under Fractional Reserve Banking

Cycles

Corollary

Assume that the buyer makes a take-it-or-leave-it offer to the seller in the DM. Let $-qu''(q)/u' = \eta$ *and* $c(q) = q$. *If* $\chi \in (0, \chi_m)$, *where*

$$\chi_m \equiv \frac{\alpha\eta(1-\sigma)}{\eta(1-\alpha\sigma)+(2-\eta)(1+i)} \tag{11}$$

then $f'(z_s) < -1$.

More cycles

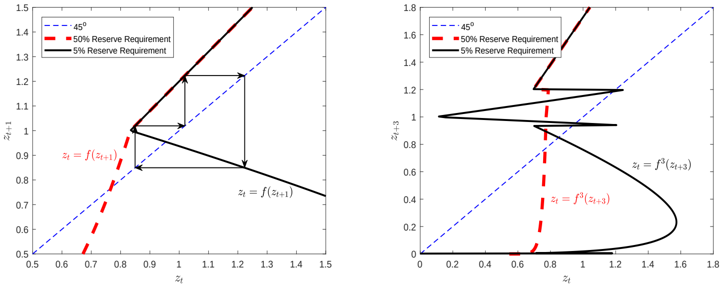


Figure 2: A Three-period Cycle under Fractional Reserve Banking

More cycles

Proposition (Three-period Monetary Cycle and Chaos)

There exists a three-period cycle with $z_1 < z_2 < z_3$ *if* $\chi \in (0, \hat{\chi}_m)$*, where*

$$\hat{\chi}_m \equiv \frac{(1-\sigma)\alpha L\left(\frac{p^*}{1+i}\right)}{(1+i)^3 - 1 - \sigma\alpha L\left(\frac{p^*}{1+i}\right)}$$

- Three-period cycle implies cycles of all periods (Sharkovskii 1964)
- Three-period cycle implies chaos (Li & Yorke 1975)

More theoretical results

Sunspot cycles

- Lowering reserve requirement can induce stochastic cycles which are independent from the fundamental.

Endogenous unsecured credit

- Allow agent can trade using endogenous credit limit arise from the voluntary repayments
- Basline model result still hold: lowering reserve requirement can induce cyclic, chaotic dynamics

Self-Fulfilling bubble and burst equilibria

- There exist endogenous bubble and burst arising from multiple equilibria when reserve requirement is lower than some threshold

Money demand

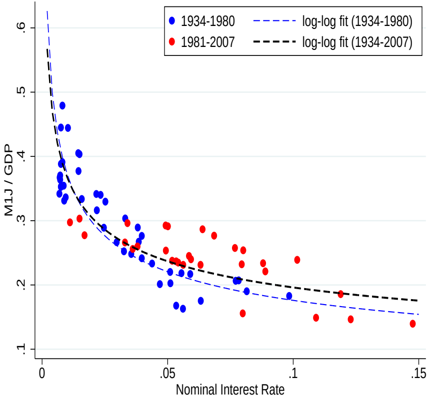


Figure 3: US M1 Money Demand and M1J

To fit money demand, I use M1J proposed by Lucas & Nicolini (2015)

Parameterization and calibrated parameter

- Buyer makes take-it-or-leave-it offer to seller in the DM.
- Matching function, $\mathcal{M}(\mathcal{B}, \mathcal{S}) = \frac{\mathcal{B}\mathcal{S}}{\mathcal{B}+\mathcal{S}}$ where $\mathcal{B}$ and $\mathcal{S}$ denotes the measure of buyers and sellers.

$$u(q) = \frac{q^{1-\eta}}{1-\eta}, \qquad c(q) = q, \qquad U(X) = B \log(X)$$

Table 1: Annual Model (1934-2007)

Parameter	Value	Target
DM utility curvature, η	0.179	elasticity of z/y wrt i
CM utility level, B	1.653	avg. z/y
fraction of buyers, σ	0.771	avg. m/y

Calibrated Examples: DM surplus

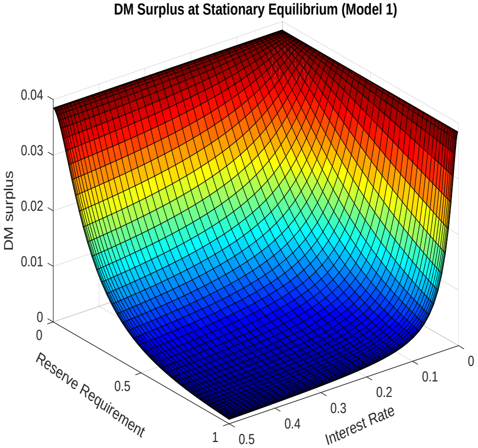


Figure 4: DM surplus at the stationary equilibrium

Calibrated examples

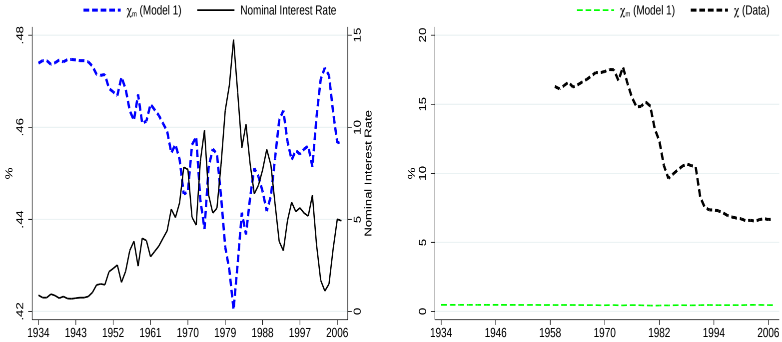


Figure 5: χ_m

Other applications

Calibrated examples based on the model with endogenous credit limit.

- Parameterization of the model using 1934 to 2007 data. Numerical examples.

News shock

- With lower reserve requirement, announcements on the future changes in monetary policy induce higher volatility.

Empirical evidence

- Cointegration between real inside money volatility, required reserve ratio, and interest rate.
- The real inside money volatility is high under the low required reserve ratio for given interest rate.

Calibrated examples | News shock | Empirical evidence | Empirical robustness 1 | Empirical robustness 2

Conclusion

- Lowering reserve requirement induce instability: more prone to exhibit endogenous cyclic, chaotic, and stochastic dynamics
- This result holds in the extended model with endogenous credit
- Lowering the reserve requirement increases the welfare at the steady state

Thank You!

Appendix

Trading Mechanisms

- **Axiom 1**: (Feasibility) $\forall z$, $0 \leq p \leq z$, $0 \leq q$.
- **Axiom 2**: (Individual Rationality) $\forall z$, $u \geq p$ and $p > c$.
- **Axiom 3**: (Monotonicity) $p(z_2) > p(z_1) \Leftrightarrow q(z_2) > q(z_1)$.
- **Axiom 4**: (Bilateral Efficiency) $\forall z$, $\nexists\ (p', q')$ with $p' < z$ such that $u'(q') - p' \geq u \circ q(z) - p(z)$ and $p' - c(q') \geq p(z) - c \geq c \circ q(z)$ with one inequality strict.

DM Trade

DM trade

Differentiating $V_{b,t}$ yields

$$\frac{\partial V_{b,t}}{\partial m} = \phi_t[\alpha\lambda(q_t) + 1] \quad (12)$$

$$\frac{\partial V_{b,t}}{\partial d} = \phi_t[-\alpha\lambda(q_t) + i_d] \quad (13)$$

$$\frac{\partial V_{b,t}}{\partial \ell} = \phi_t[\alpha\lambda(q_t) - i_l] \quad (14)$$

where $\lambda(q) = u'(q)/v'(q) - 1$ if $p^* > z$ and $\lambda(q) = 0$ if $z \geq p^*$
Differentiating $V_{s,t}$ yields

$$\frac{\partial V_{s,t}}{\partial m_t} = \phi_t, \qquad \frac{\partial V_{s,t}}{\partial d} = \phi_t(1 + i_{d,t}), \qquad \frac{\partial V_{s,t}}{\partial \ell} = -\phi_t(1 + i_{l,t}).$$

back to DM trade

Stationary Equilibrium

- Given $i \in [0, \bar{\iota})$ and $\chi \in (0, 1]$ with $\bar{\iota} = \alpha(1 - \sigma + \sigma\chi)L(0)/\chi$, an unique stationary monetary equilibrium exists satisfying

$$\chi i = (1 - \sigma + \sigma\chi)\alpha L(z_s)$$

- where $z_s = v(q_s)$.

- Simple examples for $\bar{\iota}$ under the Inada condition $u'(0) = \infty$
  - with the Nash bargaining we have $\bar{\iota} = \infty$
  - with the Kalai bargaining we have $\bar{\iota} = \theta\alpha(1 - \sigma + \sigma\chi)/\chi(1 - \theta)$

Back to Cycle

Endogenous Credit Limits

- Assume the buyer makes a take-it-or-leave-it offer to the seller in the DM and $c(q) = q$

$$V_t^b(m_t + \ell_t, 0, \ell_t) = \alpha[u(q_t) - q_t] + W_t(m_t + \ell_t, 0, \ell_t)$$

- where $q_t = \min\{q^*, \phi_t(m_t + \ell_t) + \bar{b}_t\}$.
- Given $\bar{b}_t$, solving equilibrium yields

$$z_t = \begin{cases} \frac{z_{t+1}}{1+i} \left\{ \frac{1-\sigma+\sigma\chi}{\chi} \alpha \left[u'(q_{t+1}) - 1 \right] + 1 \right\} & \text{if } w_{t+1} < q^* \\ \frac{z_{t+1}}{1+i} & \text{if } w_{t+1} \geq q^*. \end{cases} \tag{15}$$

where $w_{t+1} = z_{t+1} + \bar{b}_{t+1}$ and $z_{t+1} = (1 - \sigma + \sigma\chi)\phi_{t+1} m_{t+1} / (\sigma\chi)$

Endogenous Credit Limits

- Credit limit, $\bar{b}_t$, is determined by
- The incentive condition for voluntary repayment is

$$\underbrace{-b_t + W_t(0,0,0)}_{\text{value of honoring debts}} \geq \underbrace{(1-\mu)W_t(0,0,0) + \mu\underline{W}(0,0,0)}_{\text{value of not honoring debts}}.$$

- where the value of autarky is
  $\underline{W}(0,0,0) = \{U(X^*) - X^* + T\}/(1-\beta)$

Equilibrium

The equilibrium can be collapsed in to a dynamic system satisfying (16)-(17).

$$z_t = \begin{cases} \frac{z_{t+1}}{1+i}\left\{\frac{1-\sigma+\sigma\chi}{\chi}\alpha\left[u'(w_{t+1})-1\right]+1\right\} & \text{if } w_{t+1} < q^* \\ \frac{z_{t+1}}{1+i} & \text{if } w_{t+1} \geq q^*. \end{cases} \tag{16}$$

$$\bar{b}_t = \begin{cases} \beta\bar{b}_{t+1} + \frac{\chi\mu\sigma[-\gamma z_t + \beta z_{t+1}]}{1-\sigma+\sigma\chi} + \beta\alpha\mu\sigma S(w_{t+1}) & \text{if } w_{t+1} < q^* \\ \beta\bar{b}_{t+1} + \frac{\chi\mu\sigma[-\gamma z_t + \beta z_{t+1}]}{1-\sigma+\sigma\chi} + \beta\alpha\mu\sigma S(q^*) & \text{if } w_{t+1} \geq q^* \end{cases} \tag{17}$$

where $z_{t+1} = (1-\sigma+\sigma\chi)\phi_{t+1}m_{t+1}/(\sigma\chi)$, $w_{t+1} = z_{t+1} + \bar{b}_{t+1}$, and $S(z_{t+1}+\bar{b}_{t+1}) \equiv [u(z_{t+1}+\bar{b}_{t+1}) - z_{t+1} - \bar{b}_{t+1}]$.

Stationary Equilibrium

Let $r = 1/\beta - 1$. The debt limit at the stationary equilibrium, $\bar{b}$, is a fixed point satisfying $\bar{b} = \Omega(\bar{b})$ where

$$\Omega(\bar{b}) = \begin{cases} \frac{\mu\sigma\alpha}{r}[u(\tilde{q}) - \tilde{q}] - \frac{i\mu\sigma\chi}{1-\sigma+\sigma\chi}[\tilde{q} - \bar{b}] & \text{if } \tilde{q} > \bar{b} \geq 0 \\ \frac{\mu\sigma\alpha}{r}[u(\bar{b}) - \bar{b}] & \text{if } q^* > \bar{b} \geq \tilde{q} \\ \frac{\mu\sigma\alpha}{r}[u(q^*) - q^*] & \text{if } \bar{b} \geq q^* \end{cases} \tag{18}$$

where $\tilde{q}$ solves $u'(\tilde{q}) = 1 + i\chi/[\alpha(1-\sigma+\sigma\chi)]$.
Money and credit coexist if and only if $0 < \mu < \min\{1, \tilde{\mu}\}$, where

$$\tilde{\mu} \equiv \sigma \left\{ i\chi[(1-\sigma+\sigma\chi)/\tilde{q} - 1] + (\alpha/r)(1-\sigma+\sigma\chi)^2[u(\tilde{q})/\tilde{q} - 1] \right\}$$

since they coexist when $\bar{b} < \tilde{q}$. The DM consumption is decreasing in i in the monetary equilibrium.

Cycles with Unsecured Credit

Proposition (**Monetary Cycles with Unsecured Credit**)

There exist two period cycles of money and credit with $w_1 < q^* < w_2$ *if* $\chi \in (0, \chi_c)$, *where* $w_j = z_j + \bar{b}_j$ *and*

$$\chi_c \equiv \frac{(1-\sigma)\alpha \left[u'\left(\frac{q^*}{1+\iota}\right) - 1\right]}{(1+i)^2 - 1 - \sigma\alpha \left[u'\left(\frac{q^*}{1+\iota}\right) - 1\right]}.$$

There exist three period cycles of money and credit with $w_1 < q^* < w_2 < w_3$, *if* $\chi \in (0, \hat{\chi}_c)$, *where*

$$\hat{\chi}_c \equiv \frac{(1-\sigma)\alpha \left[u'\left(\frac{q^*}{1+\iota}\right) - 1\right]}{(1+i)^3 - 1 - \sigma\alpha \left[u'\left(\frac{q^*}{1+\iota}\right) - 1\right]}.$$

Back to other applications

Sunspot Cycles

- Consider a Markov sunspot variable $S \in \{1, 2\}$. This sunspot variable is not related with fundamentals.
- Let $Pr(S_{t+1} = 1 | S_t = 1) = \zeta_1$, $Pr(S_{t+1} = 2 | S_t = 2) = \zeta_2$
- The sunspot is realized in the CM.
- CM value function is written as

$$W_t^S(m_t, d_t, \ell_t) = \max_{X_t, H_t, \hat{m}_{t+1}} U(X_t) - H_t + \beta \left[\zeta_s G_{t+1}^S(\hat{m}_{t+1}) + (1 - \zeta_s) G_{t+1}^{-S}(\hat{m}_{t+1}) \right]$$

s.t. $\phi_t^S \hat{m}_{t+1} + X_t = H_t + T_t + \phi_t^S m_t + (1 + i_{d,t}) \phi_t^S d_t - (1 + i_{l,t}) \phi_t^S \ell_t.$

- The first order condition can be written as

$$-\phi_t^S + \beta \zeta_s G_{t+1}'^S(\hat{m}_{t+1}) + \beta(1 - \zeta_s) G_{t+1}'^{-S}(\hat{m}_{t+1}) = 0. \qquad (19)$$

$$G'^{S}_{t+1}(m^{S}_{t+1}) = \phi^{S}_{t+1}\left[\frac{1-\sigma+\sigma\chi}{\chi}\alpha L(z^{S}_{t+1}) + 1\right] \tag{20}$$

Substituting (20) into (19) and multiplying $(1-\sigma+\sigma\chi)m_{t+1}/(\sigma\chi)$ to the both sides yield

$$\begin{aligned} z^{S}_{t} =& \frac{\zeta_s z^{S}_{t+1}}{1+i}\left[\frac{1-\sigma+\sigma\chi}{\chi}\alpha L(z^{S}_{t+1}) + 1\right] \\ &+ \frac{(1-\zeta_s) z^{-S}_{t+1}}{1+i}\left[\frac{1-\sigma+\sigma\chi}{\chi}\alpha L(z^{-S}_{t+1}) + 1\right] \\ =& \zeta_s f(z^{S}_{t+1}) + (1-\zeta_s) f(z^{-S}_{t+1}) \end{aligned} \tag{21}$$

Sunspot Cycles

Definition (**Proper Sunspot Equilibrium**)

A proper sunspot equilibrium consists of the sequences of real balances $\{z_t^S\}_{t=0,S=1,2}^{\infty}$, where z_1 is not equal to z_2, and probabilities (ζ_1, ζ_2), solving (21) for all t.

Proposition (**Existence of Proper Sunspot Equilibrium**)

If $f'(z_s) < -1$, there exist (ζ_1, ζ_2), $\zeta_1 + \zeta_2 < 1$, such that the economy has a proper sunspot equilibrium in the neighborhood of z_s.

Back to other applications

Self-Fulfilling Bubble and Burst Equilibria

- Assume the buyer makes a take-it-or-leave-it offer to the seller; the DM utility function and the cost function satisfies $-qu''(q)/u'(q) = \eta$ and $c(q) = q$.
- Consider the equilibria that real balance increases above the steady state until certain time, T, and crashes to zero.
  - More specifically, consider a sequence of real balance $\{z_t\}_{t=0}^{\infty}$ with $z_T \equiv \max\{z_t\}_{t=0}^{\infty} > q^*$ (bubble) that crashes to 0 (burst) as $t \to \infty$, where $T \geq 1$ and $z_T > z_0$.

Definition (**Self-Fulfilling Bubble and Burst Equilibria**)

For initial level of real balance $z_0 > 0$, a self-fulfilling bubble and burst is a set of sequence $\{z_t\}_{t=0}^{\infty}$ satisfying (22)

$$z_t = \frac{z_{t+1}}{1+i}\left[\frac{1-\sigma+\sigma\chi}{\chi}\alpha[u'(z_{t+1})-1]+1\right] \tag{22}$$

where $0 < z_s < z_T$, $\lim_{t\to\infty} z_t = 0$, $z_T = \max\{z_t\}_{t=0}^{\infty}$ with $T \geq 1$.

Back to other applications

Self-Fulfilling Bubble and Burst Equilibria

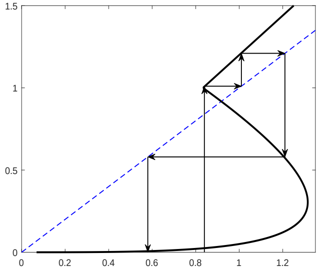


Figure 6: Bubble and Burst Equilibria

- When $z_s > \bar{z}$, where $\bar{z}$ solves $f'(\bar{z}) = 0$, there exist multiple equilibria.
- Then, if $f(\bar{z}) \geq q^*$, the self-fulfilling bubble and burst equilibria exist.

Self-Fulfilling Bubble and Burst Equilibria

Proposition (**Existence of Self-Fulfilling Bubble and Burst Equilibria**)

There exist self-fullfilling bubble and burst equilibria, $\{z_t\}_{t=0}^{\infty}$ *if*

$$0 < \chi < \min\left\{\frac{(1-\sigma)\alpha\eta(1+i)}{(1-\eta)^2 q^* + (1+i)[(1-\eta)(3+i-\eta)-\alpha\sigma\eta]}, Q(i)\right\}$$

where $Q(i) = \frac{(1-\sigma)\alpha\eta}{2+i(2-\eta)-\alpha\sigma\eta}$

Back to other applications

Money Demand

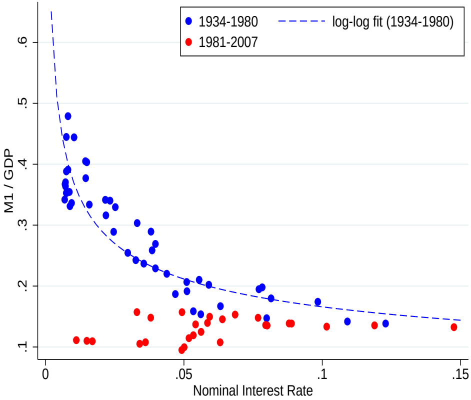


Figure 7: US M1 Money Demand

Two Different Strategy

Divide into two subperiod

- Ireland (2009, AER) and Alvarez & Lippi (2014, AEJ:macro), Berentsen et al. (2011, AER) and Berentsen et al. (2015, JMCB)

Using M1J

- Wang et al. (2020, IER) and Bethune et al. (2020, RES). Lucas & Nicolini (2015, JME)

Adapt both

- Model 1: calibrate the model without unsecured credit using M1J
- Model 2: calibrate the model with unsecured credit using M1 and unsecured credit assuming there were structural break at 1980.

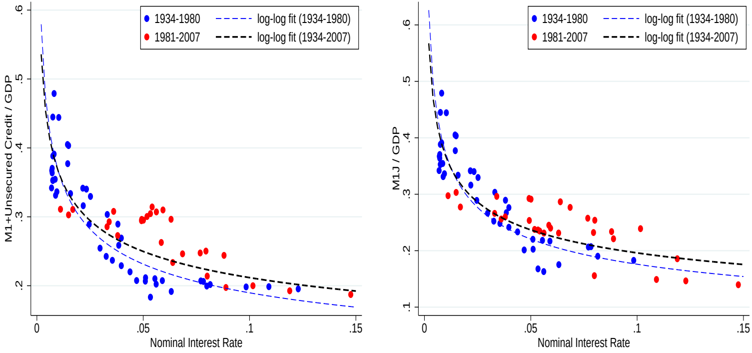


Figure 8: US Money Demand and Credit

Unsecured Credit Growth

Institutional Changes and Unsecured Credit Growth

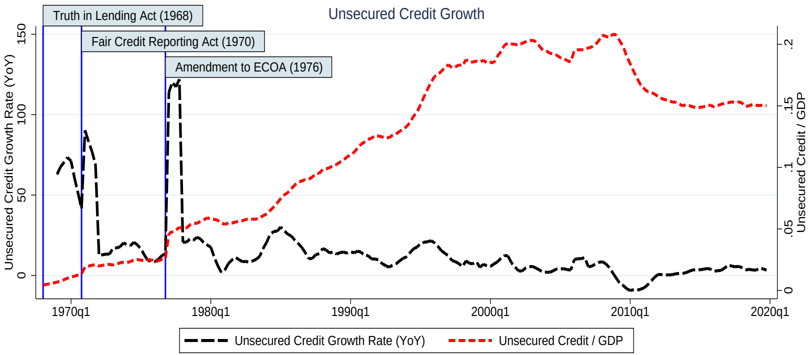


Figure 9: Institutional Changes and Unsecured Credit Growth

Back to the calibration

Parameterization and Calibrated Parameter

- Buyer makes take-it-or-leave-it offer to seller in the DM.
- Matching function, $\mathcal{M}(\mathcal{B}, \mathcal{S}) = \frac{\mathcal{B}\mathcal{S}}{\mathcal{B}+\mathcal{S}}$ where $\mathcal{B}$ and $\mathcal{S}$ denotes the measure of buyers and sellers.

$$u(q) = \frac{q^{1-\eta}}{1-\eta}, \qquad c(q) = q, \qquad U(X) = B \log(X)$$

Table 2: Annual Model (1934-2007)

Parameter	Model 1	Model 2	Target
DM utility curvature, η	0.179	0.129	elasticity of z/y wrt i
CM utility level, B	1.653	0.952	avg. z/y
fraction of buyers, σ	0.771	0.790	avg. m/y
monitoring probability, μ	-	0.402	avg. b/y

Calibrated Examples

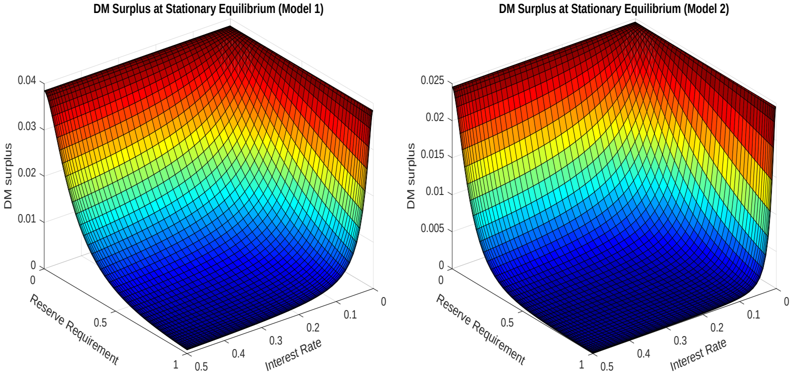


Figure 10: DM surplus at the stationary equilibrium

Calibrated Examples

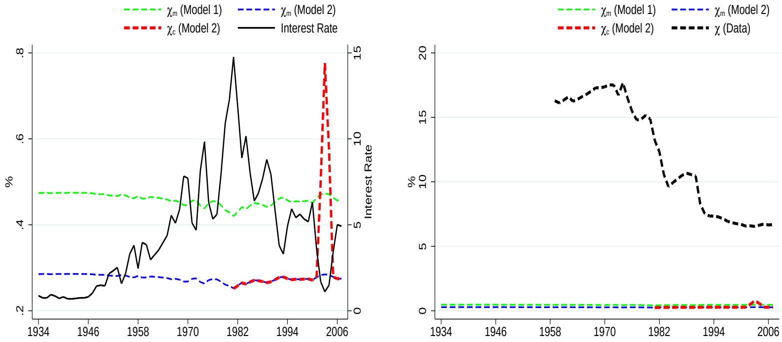


Figure 11: χ_m and χ_c

Back to other applications

News Shock

$$z_T = f_T(z_T), \quad z_{T-1} = f_0(z_T), \quad z_{T-2} = f_0(z_{T-1}), \quad \ldots \quad z_0 = f_0(z_0)$$

Let equations (16) and (17) be $z_t = \Phi(z_{t+1}, \bar{b}_{t+1})$ and $\bar{b}_t = \Gamma(z_{t+1}, \bar{b}_{t+1})$. The transitional dynamics of the equilibrium with unsecured credit also can be solved by backward induction.

$$z_T = \Phi_T(z_T, \bar{b}_T), \quad z_{T-1} = \Phi_0(z_T, \bar{b}_T), \quad z_{T-2}, \quad \ldots \quad z_0 = \Phi_0(z_0, \bar{b}_0)$$
$$\bar{b}_T = \Gamma_T(z_T, \bar{b}_T), \quad \bar{b}_{T-1} = \Gamma_0(z_T, \bar{b}_T), \quad \bar{b}_{T-2}, \quad \ldots \quad \bar{b}_0 = \Gamma_0(z_0, \bar{b}_0)$$

Table 3: Quarterly Model (1934-2007)

Parameter	Model 1	Model 2	Target
DM utility curvature, η	0.179	0.129	elasticity of z/y wrt i
CM utility level, B	0.007	0.024	avg. z/y
fraction of buyers, σ	0.805	0.917	avg. m/y
monitoring probability, μ	-	0.474	avg. b/y

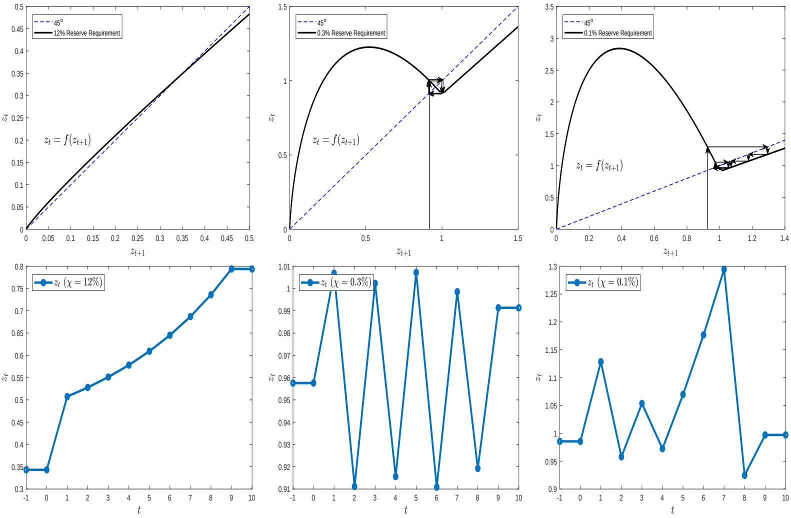


Figure 12: Phase Dynamics and Transition Paths for Known Policy Change: Model 1

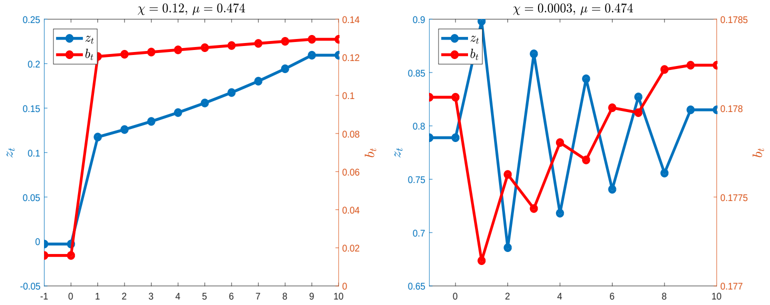


Figure 13: Transition Paths for Known Policy Change: Model 2

Back to other applications

Empirical Evaluation: Inside Money Volatility

Empirical Evaluation: Inside Money Volatility

- Do data show high volatility of real balance of inside money under lower reserve requirement?

Data

- required reserve ratio is calculated as (required reserves)/(total checkable deposit): χ
- cyclical volatility in quarter t is calculated as the standard deviation of filtered log real total checkable deposit during a 41-quarter (10-year) window centered around quarter t: σ_t^{Roll}
  1. quarterly observations are averaged for each year
  2. the Hodrick-Prescott (HP) filter with 1600 smoothing parameter
  3. real total checkable deposit is caluculated using CPI
  4. sensitivity analysis using Core CPI, PCE, Core PCE
- federal funds rate: `ffr`
- sample period: 1960Q1-2017Q4 $\Rightarrow$ 1965-2012

Empirical Evaluation: Inside Money Volatility

- Unit test fail to reject the nonstationarity of χ, σ_t^{Roll}, and `ffr` $\Rightarrow$ Spurious regression?
- Johansen test suggests that χ, σ_t^{Roll}, and `ffr` are cointegrated.
- With the cointegration relationship, we may not have to worry about a spurious relationship.
- Estimate cointegrating relationship using canonical cointegrating regression (CCR) and Fully Modified OLS (FMOLS)

Empirical Evaluation: Inside Money Volatility

Table 4: Empirical Evaluation

(a) Unit Root Test

	Phillips-Perron test		ADF test
	$Z(\rho)$	$Z(t)$	$Z(t)$ w/ lag 1
`ffr`	-6.766	-1.704	-2.362
χ	-1.518	-1.199	-1.363
σ_t^{Roll}	-4.708	-2.191	-2.090
Δ`ffr`	-28.373***	-5.061***	-6.357***
$\Delta\chi$	-31.783***	-4.794***	-3.682***
$\Delta\sigma_t^{Roll}$	-24.905***	-3.416**	-2.942**

(b) Johansen Test for Cointegration

Max rank	$\lambda_{trace}(r)$	5% CV	1% CV
0	35.6880	29.68	35.65
1	**10.6820**	15.41	20.04
2	4.5391	3.76	6.65

Max rank	$\lambda_{max}(r, r+1)$	5% CV	1% CV
0	25.0060	20.97	25.52
1	**6.1429**	14.07	18.63
2	4.5391	3.76	6.65

Empirical Evaluation: Inside Money Volatility

Table 5: Empirical Evaluation

Table 6: Effect of Require Reserve Ratio

	OLS (1)	CCR (2)	FMOLS (3)
χ	-0.283***	-0.245***	-0.211***
	(0.031)	(0.002)	(0.003)
`ffr`		-0.109***	-0.248***
		(0.002)	(0.003)
Constant	0.074***	0.074***	0.078***
	(0.004)	(0.000)	(0.000)
Obs.	49	49	49
R^2	0.706	0.637	0.144

Back to other applications

Empirical Appendix

Table 7: Effect of Require Reserve Ratio: Robustness Check

(a) Benchmark: CPI

	OLS (1)	CCR (2)	FMOLS (3)
χ	-0.283***	-0.245***	-0.211***
	(0.031)	(0.002)	(0.003)
ffr		-0.109***	-0.248***
		(0.002)	(0.003)
Constant	0.074***	0.074***	0.078***
	(0.004)	(0.000)	(0.000)
Obs.	49	49	49
R^2	0.706	0.637	0.144

(b) Core CPI

OLS (1)	CCR (2)	FMOLS (3)
-0.267***	-0.221***	-0.192***
(0.027)	(0.003)	(0.003)
	-0.125***	-0.248***
	(0.003)	(0.004)
0.070***	0.071***	0.074***
(0.004)	(0.000)	(0.000)
49	49	49
0.734	0.663	0.133

(c) PCE

	OLS (1)	CCR (2)	FMOLS (3)
χ	-0.306***	-0.227***	-0.189***
	(0.029)	(0.004)	(0.005)
ffr		-0.187***	-0.350***
		(0.004)	(0.005)
Constant	0.074***	0.075***	0.079***
	(0.004)	(0.000)	(0.000)
Obs.	49	49	49
R^2	0.746	0.664	0.121

(d) Core PCE

	OLS (1)	CCR (2)	FMOLS (3)
χ	-0.307***	-0.220***	-0.182***
	(0.027)	(0.005)	(0.005)
ffr		-0.207***	-0.362***
		(0.004)	(0.006)
Constant	0.073***	0.073***	0.077***
	(0.004)	(0.000)	(0.001)
Obs.	49	49	49
R^2	0.769	0.680	-0.042

Empirical Appendix

Table 9: Johansen Test for Cointegration: Robustness Check

(a) Benchmark: CPI

Max rank	$\lambda_{trace}(r)$
0	35.6880
1	**10.6820**
2	4.5391

Max rank	$\lambda_{max}(r, r+1)$
0	25.0060
1	**6.1429**
2	4.5391

(b) Core CPI

Max rank	$\lambda_{trace}(r)$	5% CV	1% CV
0	35.1449	29.68	35.65
1	**10.0645**	15.41	20.04
2	4.2011	3.76	6.65

Max rank	$\lambda_{max}(r, r+1)$	5% CV	1% CV
0	25.0804	20.97	25.52
1	**5.8635**	14.07	18.63
2	4.2011	3.76	6.65

(c) PCE

Max rank	$\lambda_{trace}(r)$
0	35.3667
1	**9.8942**
2	3.9605

Max rank	$\lambda_{max}(r, r+1)$
0	25.4725
1	**5.9337**
2	3.9605

(d) Core PCE

Max rank	$\lambda_{trace}(r)$	5% CV	1% CV
0	35.0280	29.68	35.65
1	**9.3450**	15.41	20.04
2	3.6465	3.76	6.65

Max rank	$\lambda_{max}(r, r+1)$	5% CV	1% CV
0	25.6830	20.97	25.52
1	**5.6986**	14.07	18.63
2	3.6465	3.76	6.65

Empirical Appendix

Table 10: Unit Root Test: Robustness Check

(a) Benchmark: CPI

	Phillips-Perron test		ADF test
	$Z(\rho)$	$Z(t)$	$Z(t)$ w/ lag 1
σ_t^{Roll}	-4.708	-2.191	-2.090
$\Delta\sigma_t^{Roll}$	-24.905***	-3.416**	-2.942**

(b) Core CPI

	Phillips-Perron test		ADF test
	$Z(\rho)$	$Z(t)$	$Z(t)$ w/ lag 1
σ_t^{Roll}	-4.681	-2.189	-1.978
$\Delta\sigma_t^{Roll}$	-24.758***	-3.509***	-2.942***

(c) PCE

	Phillips-Perron test		ADF test
	$Z(\rho)$	$Z(t)$	$Z(t)$ w/ lag 1
σ_t^{Roll}	-4.329	-2.038	-2.047
$\Delta\sigma_t^{Roll}$	-23.691***	-3.330***	-2.842**

(d) Core PCE

	Phillips-Perron test		ADF test
	$Z(\rho)$	$Z(t)$	$Z(t)$ w/ lag 1
σ_t^{Roll}	-4.076	-1.954	-1.930
$\Delta\sigma_t^{Roll}$	-22.826***	-3.296**	-2.768**

Empirical Appendix

Table 11: Empirical Evaluation: Robustness Check (Quarterly)
(a) Unit Root Test

	Phillips-Perron test		ADF test
	$Z(\rho)$	$Z(t)$	$Z(t)$ w/ lag 1
ffr	-8.900	-1.989	-2.219
χ	-1.263	-1.092	-1.150
σ_t^{Roll}	-3.946	-2.372	-2.227
Δffr	-136.820***	-10.679***	-10.179***
$\Delta\chi$	-160.164***	-12.130***	-9.804***
$\Delta\sigma_t^{Roll}$	-40.319***	-4.515**	-5.627**

(b) Johansen Test for Cointegration

Max rank	$\lambda_{trace}(r)$	5% CV	1% CV
0	35.5243	29.68	35.65
1	**15.2586**	15.41	20.04
2	4.0275	3.76	6.65

Max rank	$\lambda_{max}(r, r+1)$	5% CV	1% CV
0	20.2657	20.97	25.52
1	11.2311	14.07	18.63
2	4.0275	3.76	6.65

Back to other applications

Empirical Appendix

Table 13: Empirical Evaluation: Robustness Check (Quarterly)

Table 14: Effect of Require Reserve Ratio

	OLS	CCR	FMOLS
	(1)	(2)	(3)
χ	-0.286***	-0.405***	-0.464***
	(0.016)	(0.000)	(0.000)
`ffr`		-0.120***	-0.279***
		(0.000)	(0.000)
Constant	0.074***	0.080***	0.077***
	(0.002)	(0.000)	(0.000)
Obs.	192	192	192
R^2	0.719	0.403	0.081

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